

$$\frac{d^n y}{dx^n} = \int e^x$$

$$C \int e^x = C e^x + C$$

Inverse Trig Functions

None of the 6 trig functions has an inverse because none are one to one on their domains, we will be redefining their domains so that we can find an inverse function.

$$8000 \quad 20,000$$

$$67^\circ \quad 69^\circ$$

$$t = 4\pi \quad \tau = \pi = 4$$

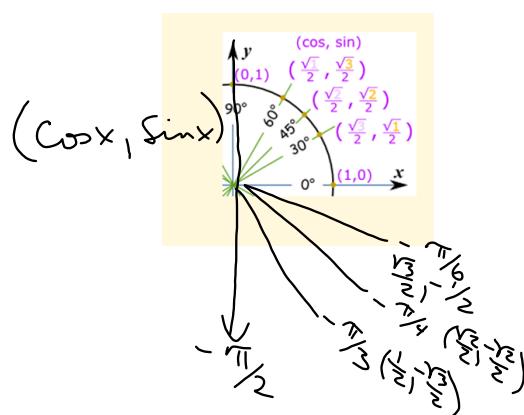
$$n = 365 \quad 365(\pi) \quad n = 365$$

$$A = P \left(1 + \frac{0.6}{365}\right)$$

Definitions of Inverse Trig Functions

Function	Domain	Range
$y = \arcsin x$ iff $\sin y = x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \arccos x$ iff $\cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \arctan x$ iff $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \text{arc cot } x$ iff $\cot y = x$	$-\infty < x < \infty$	$0 < y < \pi$
$y = \text{arc sec } x$ iff $\sec y = x$	$ x \geq 1$	$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$
$y = \text{arc csc } x$ iff $\csc y = x$	$ x \geq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$

Note: The term $\arcsin x$ is read as the arcsine of x or the angle whose sine is x . $\sin^{-1} x$ is another notation



Examples:

Evaluate each of the following

$$\arcsin\left(-\frac{1}{2}\right) \Rightarrow \sin y = -\frac{1}{2} \text{ so in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y = -\frac{\pi}{6}$$

$$\arccos(0) \Rightarrow \cos y = 0, \text{ so in } [0, \pi], y = \frac{\pi}{2}$$

$$\arctan\sqrt{3} \Rightarrow \tan y = \sqrt{3}, \text{ so in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), y = \frac{\pi}{3}$$

$$\arcsin(0.3) \Rightarrow \sin^{-1}(0.3) \approx 0.305$$

Remember, inverse functions have the properties:

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x$$

Properties of Inverse Trig functions

If $-1 \leq x \leq 1$ and $-\pi/2 \leq y \leq \pi/2$, then
 $\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$

If $-\pi/2 < y < \pi/2$, then
 $\tan(\arctan x) = x$ and $\arctan(\tan y) = y$

If $|x| \geq 1$ and $0 \leq y < \pi/2$ or $\pi/2 < y < \pi$, then
 $\sec(\operatorname{arcsec} x) = x$ and $\operatorname{arcsec}(\sec y) = y$

Solving an equation

$$\arctan(2x-3) = \frac{\pi}{4} \quad \tan \frac{\pi}{4} = 2x-3$$

$$\tan[\arctan(2x-3)] = \tan \frac{\pi}{4}$$

$$2x-3=1 \\ x=2$$

Solving using right triangles

angle
 $y = \arcsin x$ where $0 < y < \pi/2$
 find $\cos y$
 $\sin y = x$ or $\frac{x}{1}$ opp hyp
 $\cos y = \cos(\arcsin)x = \frac{\text{adj}}{\text{hyp}} = \sqrt{1-x^2}$

$$x^2 + b^2 = 1^2 \\ b^2 = 1 - x^2 \\ b = \sqrt{1-x^2}$$

$y = \operatorname{arcsec}(\sqrt{5}/2)$ find $\tan y$
 $\sec y = \sqrt{5}/2$ opp adj
 $\tan y = \tan(\operatorname{arcsec}(\sqrt{5}/2)) = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$

$$\sqrt{5}^2 - 2^2 = a^2 \\ a^2 = 5 - 4 \\ a = 1$$

Derivatives of Inverse trig functions

Let u be a differentiable function of x .

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arc cot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arc sec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}[\operatorname{arc csc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Notice the similarity in the pairs?

example

$$\frac{d}{dx}[\arcsin 2x] = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$$

$$\frac{d}{dx}[\arctan 3x] = \frac{3}{1+9x^2}$$

$$\frac{d}{dx}[\arcsin \sqrt{x}] = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$\frac{d}{dx}[\operatorname{arc sec} e^{2x}] = \frac{1}{|e^{2x}| \sqrt{e^{4x}-1}} = \frac{2}{2\sqrt{e^{4x}-1}}$$

Find y'

$$y = \arcsin x \quad y' = \frac{1}{\sqrt{1-x^2}}$$

$$y' = 2\sqrt{1-x^2}$$

What are the extrema for

$$y = [\arctan x]^2$$

$$y' = 2(\arctan x) \frac{1}{1+x^2} = \frac{2(\arctan x)}{1+x^2}$$

$$\frac{2(\arctan x)}{1+x^2} = 0$$

$$2(\arctan x) = 0$$

$$\arctan x = 0$$

$$\tan y = 0$$

$$y = 0$$

$$(-\frac{\pi}{2}, 0)(0, \frac{\pi}{2})$$

$$y < 0 \quad y > 0$$

Minimum value at 0

Assignment

P 377 Day 1 # 3-45 by 3's,
Day 2 48-65 by 3's, 71-77 odd, 81, 91