

Riemann Sums and Definite Integrals

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x; \Delta x = \frac{b-a}{n}$$

$\|\Delta\|$ is the length of the largest subinterval in $[a,b]$.

$\|\Delta\|$ is the norm of $[a,b]$.

If subintervals are all of equal length, the partition is regular and $\|\Delta\| = \Delta x = \frac{b-a}{n}$

$$\|\Delta\| \rightarrow 0 \Rightarrow n \rightarrow \infty$$

Definite Integral

f is defined on $[a, b]$ and the limit below exists

$$\lim_{\|\Delta\| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) dx$$

a is the lower limit of integration

b is the upper limit of integration

c_i is any point on the i^{th} subinterval

Δx_i is the width of the i^{th} subinterval

A definite integral is a number. (+, -, 0)

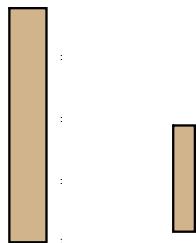
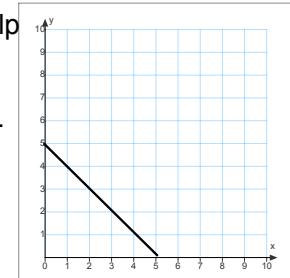
$$\begin{aligned}
 \int_{-2}^1 2x dx &= \lim_{\|\Delta\| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \\
 \Delta x &= \frac{b-a}{n} = \frac{1-(-2)}{n} = \frac{3}{n} & c_i = a + \Delta x i = -2 + \frac{3}{n} i \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n 2(-2 + \frac{3}{n} i) \left(\frac{3}{n}\right) \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (-4 + \frac{6i}{n}) \left(\frac{3}{n}\right) \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{-12}{n} + \frac{18i}{n^2} \right) \\
 &= \lim_{n \rightarrow \infty} \left[\sum_{i=1}^n \left(-\frac{12}{n} \right) + \sum_{i=1}^n \left(\frac{18i}{n^2} \right) \right] \\
 &= \lim_{n \rightarrow \infty} \left[-\frac{12}{n} (n) + \frac{18}{n^2} \left(\frac{n(n+1)}{2} \right) \right] \\
 &= \lim_{n \rightarrow \infty} \left[-12 + \frac{18}{2} \left(\frac{n(n+1)}{n} \right) \right] = -12 + 9(1) = -3
 \end{aligned}$$

$$\int_1^3 x^2 dx$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x$$

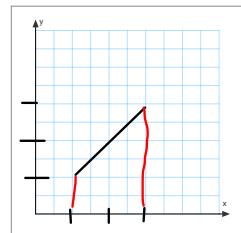
You can also use a geometric formula to help find the area under the curve if your region is a known geometry shape.

$$\int_0^5 (5-x)dx$$



$$\int_1^3 x dx$$

= area of trapezoid



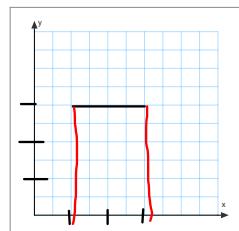
Definitions

$$1. \int_a^a f(x)dx = 0$$

$$2. \int_b^a f(x)dx = - \int_a^b f(x)dx$$

$$\text{ex } \int_5^0 (5-x)dx = - \int_0^5 (5-x)dx$$

$$\int_1^3 3dx$$



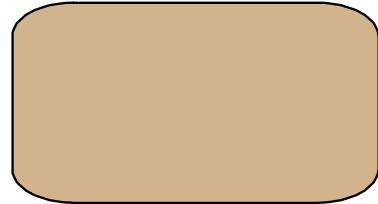
Properties

$$1. \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$2. \int_a^b kf(x)dx = k \int_a^b f(x)dx$$

$$3. \int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_1^3 (-x^2 + 4x - 3)dx$$



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Given: $\int_0^3 f(x)dx = 4$

$$\int_3^6 f(x)dx = -1$$

Find (a) $\int_0^6 f(x)dx =$

Find (b) $\int_6^3 f(x)dx =$

Find (c) $\int_3^3 f(x)dx =$

Find (d) $\int_3^6 -5f(x)dx =$

Try This

