Sigma Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_x + a_3 + \ldots + a_n$$

$$\sum_{i=1}^{5} 2i = 2(1) + 2(2) + 2(3) + 2(4) + 2(5)$$

$$= 2 + 4 + 6 + 8 + 10 = 30$$

$$\sum_{k=3}^{7} k^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2$$

$$= 9 + 16 + 25 + 36 + 49 =$$

$$\sum_{i=1}^{4} 5 = 5 + 5 + 5 + 5 = 4(5)$$

General ideas

$$\sum_{i=1}^{n} c = n(c) \ (lower \ \ bound = 1)$$

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$$

$$\sum (a_i \pm b_i) = \sum a_i \pm \sum b_i$$

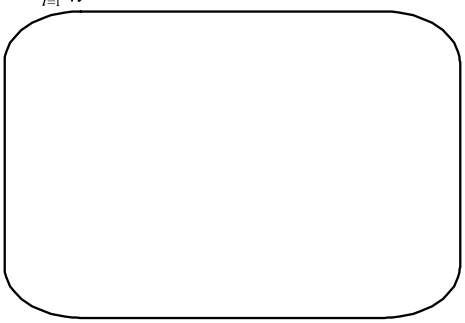
Need to Know!!

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

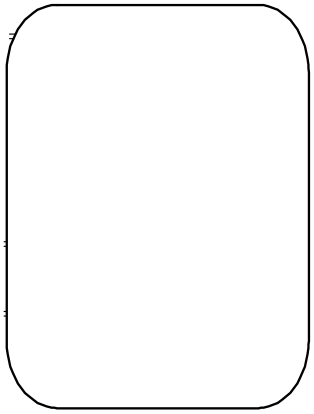
$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} \frac{i^2}{n^3} = \sum_{i=1}^{n} \frac{1}{n^3} i^2$$



Couple the idea of summation with limits

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2}{n^3}=\sum_{i=1}^{n}\frac{1}{n^3}i^2$$



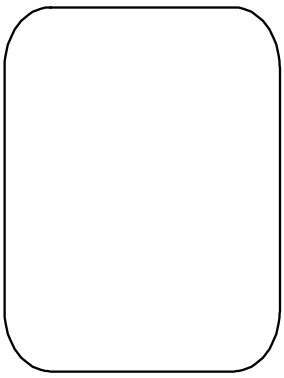
Major ideas:

Factor out all of the constants

Separate all of our fractions

Techniques

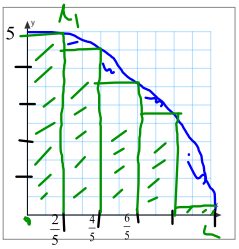
$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{(i-1)^2}{n^3}$$



Area's

We are going to compare the area's of inscribed versus circumscribed polygons to determine the area under a curve

$f(x) = -x^2 + 5$



5 rectangles

6.48 is less than the area of the region

Area of Rectangles

$= \sum_{i=1}^{5} f(\frac{2}{5}i)(\frac{2}{5})$

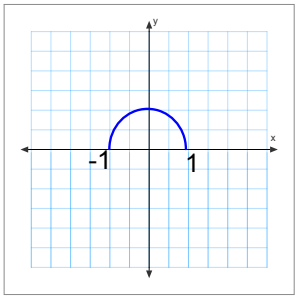
Let f be continuous and nonnegative on [a, b]. The area of the region bounded by the graph of f, the x-axis and the vertical lines x = a, x = b is

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$f(x) = 1 - x^2, from\ [-1,1]$

$\Delta x = \frac{1-(-1)}{n} = \frac{2}{n}$



Area $= \lim_{n\to\infty} f(c_i)\Delta x$

$= \sum\left(\frac{8i}{n^2} - \frac{8i^2}{n^3}\right)$

Practice: $\lim_{n\to\infty} \sum_{i=1}^{n} (1 + \frac{2i}{n})^3 (\frac{2}{n})$

50. $y = x^2 + 1, [0, 3]$ Find the area

64. $f(x) = x^2 + 4x$ Find area using midpoints and n=4