Anti-differation and
Indefinite Integration

An antiderivative is the inverse of a derivative.

Consider the functions

$F(x) = x^3; \quad F'(x) = 3x^2$

$F(x) = x^5 + 4; \; F'(x) = 5x^4$

$F(x) = x^5 - 7; \; F'(x) = 5x^4$

Antiderivatives are a family of functions that come from a given derivative

Differential Equations	General Solutions
$y' = 2$	$y = 2x + c$
$y' = 4x^3$	$y = x^4 + c$

The question we ask is
"What is y if dy/dx = f(x)"

$$dy = f(x)dx$$

$$\int dy = \int f(x)dx = F(x) + c$$

Integrand — Variable of integration — constant of Integration

Rules of Integration

$$\int kf(x)dx = k\int f(x)dx$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int dx = x + c$$

$$\int kdx = kx + c \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Examples

$$\int x^4 dx = \frac{1}{5}x^5 + c$$

$$\int 5x^2 dx = 5\int x^2 dx =$$

$$5(\frac{1}{3})x^3 + c = \frac{5}{3}x^3 + c$$

$$\int (3x^4 - 5x^2 + x)dx$$

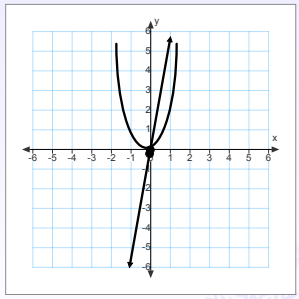
$$\int \frac{x+1}{\sqrt{x}}dx =$$

Initial Conditions

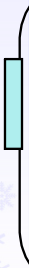
$f'(x) = 5x$

$f(x) = \int 5xdx$

$f(2) = 11$



Find y given y' = 2(x -1) and (3,2) is on the graph.



Solve: Try This

$f''(x) = x^2;\ f'(0) = 6;\ f(0) = 3$

ans:

The rate of growth of a population of bacteria is proportional to the square root of t where P is population size and t if time in days. The initial size of the population is 500 and after one day the population is 600. Estimate the population after 7 days

$\frac{dP}{dt} = k\sqrt{t}$

$P(0) = 500;\ P(1) = 600$