Continuity and one-sided limits

Objectives

- Determine the continuity at a point and continuity on an open interval
- Determine one-sided limits and continuity on a closed interval
- Use properties of continuity
- Understand and use the Intermediate Value Theorem (IVT)

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Definition of continuity

Continuity at a Point: A function f is continuous at c if the following conditions are met

1. $f(c)$ *is defined.*
2. $\lim\limits_{x\to c} f(x)$ *exists*
3. $\lim\limits_{x\to c} f(x) = f(c)$

Continuity on an open interval: A function is continuous on an open interval (a,b) if it is continuous at each point in the interval. A function tat is continuous on the entire real line $(-\infty,\infty)$ is everywhere continuous.

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Functions can have the following discontinuities:

- Removable: Graph has a hole at some value of x, or the graph has a hole at some value of x on the continuous part and a point not on the continuous part.
- Non-removable: A jump in the graph, usually seen in a piecewise function, or a graph with an asymptote

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We look at one-sided limits when our continuous function is on a closed interval.

Limit from the left

$$\lim_{x \to c^-} f(x) = L$$

Limit from the right

$$\lim_{x \to c^+} f(x) = L$$

Example

$$\lim_{x \to 0^+} \sqrt[n]{x} = 0, \; n \; is \; an \; even \; integer$$

because the function is continuous form [0,∞)

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The existence of a limit

Let f be a function and let c and L be real numbers. The limit of f(x) as x approaches c is L if and only if

$$\lim_{x\to c^-} f(x) = L \quad \text{and} \quad \lim_{x\to c^+} f(x) = L$$

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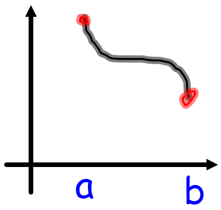
Definition of continuity on a closed Interval

A function f is continuous on the closed interval [a,b] if it is continuous on the open interval (a,b) and

$$\lim_{x\to a^+} f(x) = f(a) \quad \text{and} \quad \lim_{x\to b^-} f(x) = f(b)$$

The function f is continuous from the right at a and continuous from the left at b



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Properties of Continuity

If b is a real number and f and g are continuous at x = c, then the following functions are also continuous at c

1. Scalar multiple: bf
2. Sum and Difference: $f \pm g$
3. Product: fg
4. Quotient: f/g, if $g(c) \neq 0$

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Functions that are continuous at every point in their domains:

Polynomial Functions
Rational Functions
Radical Functions
Trigonometric functions

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Continuity of a composite function

If g is continuous at c and f is continuous at g(c), then the composite function given by $(f \circ g)(x)=f(g(x))$ is continuous at c.

That is: $$\lim_{x \to c} f(g(x)) = f(g(c))$$

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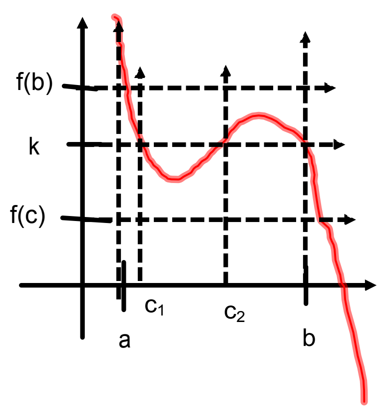
Intermediate Value Theorem

If f is continuous on the closed interval [a,b] and k is any number between f(a) and f(b), then there is at least one number c in [a,b] such that f(c)=k



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