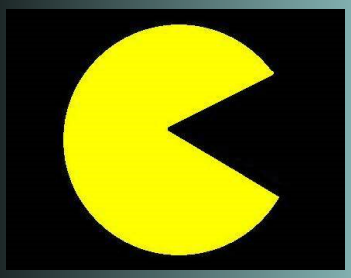
8.1 Measuring Angles & Arcs

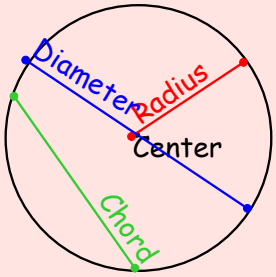


OBJECTIVE

Recognize major arcs, minor arcs, semicircles, and central angles and their measures.

New Vocabulary

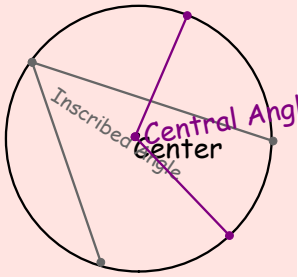
arc
Central angle
Inscribed Angles
Intercepted Arc
minor arc
major arc
semicircle



Segments

360°

Angles



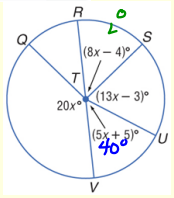
Measures of Central Angles

KEY CONCEPT — *Sum of Central Angles*

Words The sum of the measures of the central angles of a circle with no interior points in common is 360.

Example: $m\angle 1 + m\angle 2 + m\angle 3 = 360$

ALGEBRA $\overline{RV}$ **is a diameter of** $\odot T$. **Find** $m\angle RTS$.



The sum of the measures of $\angle RTS$, $\angle STU$, and $\angle UTV$ is 180.

$(8x-4)+(13x-3)+(5x+5)=180$ Substuon

$26x-2=180$ Simplify.

$26x=182$ Add 2 to each side.

$x=7$ Divide each side by 26.

Use the value of x to find $m\angle RTS$.

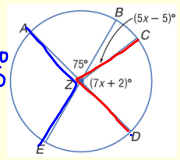
$m\angle RTS = 8x-4$ Given

$= 8(7)-4 \text{ or } 52$ Substuon

Answer: $m\angle RTS = 52$

Now, you try!

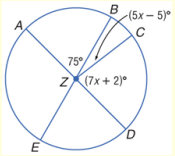
A. ALGEBRA Refer to $\odot Z$. Find $m\angle CZD$.



$m\angle CZD = 65°$

Now, you try again!

B. ALGEBRA Refer to $\odot Z$. Find $m\angle BZC$.



Angle/Arc Relationship

Central Angles

$m\angle ADC = m\overset{\frown}{AC}$

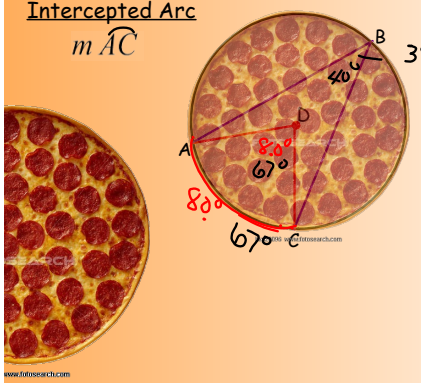
Inscribed Angles

$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

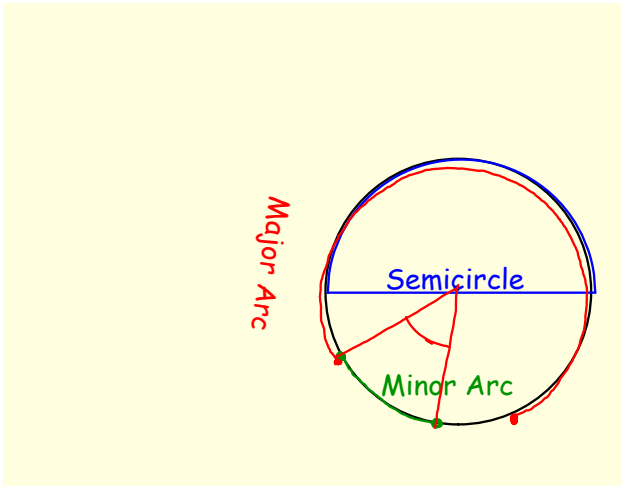
$2\, m\angle ABC = m\overset{\frown}{AC}$

Intercepted Arc

$m\overset{\frown}{AC}$



Arc Length



KEY CONCEPT *Arcs of a Circle*

Type of Arc:	minor arc	major arc	semicircle
Definition:	an arc that measures less than 180°	an arc that measures greater than 180°	an arc that measures 180°
Example:	$\overset{\frown}{AC}$	$\overset{\frown}{DFE}$	$\overset{\frown}{JKL}$ $\overset{\frown}{JML}$
Named:	usually by the letters of the two endpoints $\overset{\frown}{AC}$	by the letters of the two endpoints and another point on the arc $\overset{\frown}{DFE}$	by the letters of the two endpoints and another point on the arc $m\overset{\frown}{JML}$ and $\overset{\frown}{JKL}$
Arc Degree Measure Equals:	the measure of the central angle $m\angle ABC = 110$, so $m\overset{\frown}{AC} = 110$	360 minus the measure of the minor arc with the same endpoints $m\overset{\frown}{DFE} = 360 - m\overset{\frown}{DE}$ $m\overset{\frown}{DFE} = 360 - 60$ or 300	360 ÷ 2 or 180 $m\overset{\frown}{JML} = 180$ $m\overset{\frown}{JKL} = 180$

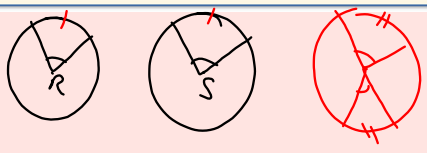
THEOREM 10.1

In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

POSTULATE 10.1 *Arc Addition Postulate*

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example: In $\odot S$, $m\overset{\frown}{PQ} + m\overset{\frown}{QR} = m\overset{\frown}{PQR}$.



Find x, such that RV is a diameter. Find all arc and angle measures.

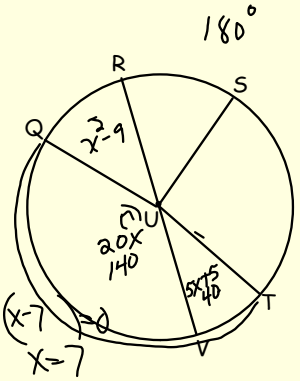
$\angle QUR = (x^2 - 9)°$

$\angle RUS = (x^2 + 3)°$

$\angle SUT = (13x - 3)°$

$\angle TUV = (5x + 5)°$

$\angle VUQ = 20x°$



Find the length of $\overset{\frown}{VQ}$.

$x^2 - 9 + 20x = 180$

$x^2 + 20x - 189 = 0$

-189

$-9 \quad 21$

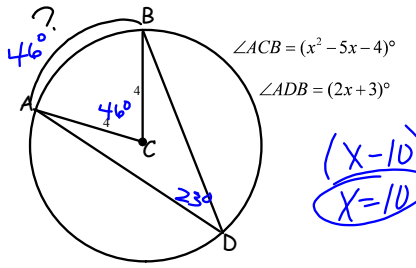
$3 \; 3 \; 3 \; 7$

$27 \quad -7$

$(x+27)(x-7) = 0$

$x = -7$

Find the length of $\overset{\frown}{AB}$.

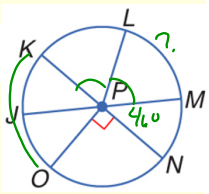


$(x-10)(x+1)=0$

$x=10$ $x=-1$

Measures of Arcs

A. In $\odot P$, $m\angle NPM = 46$, $\overline{PL}$ **bisects** $\angle KPM$, **and** $\overline{OP} \perp \overline{KN}$. **Find** $m\overset{\frown}{OK}$.



$180 - 46 = 134$

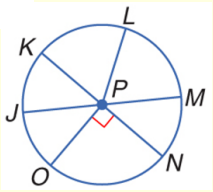
$\frac{134}{2}$

$67°$

$\overset{\frown}{OK}$ is a minor arc, so	$m\overset{\frown}{OK} = m\angle KPO$.
$\overset{\frown}{KON}$ is a semicircle.	
$m\overset{\frown}{ON} = m\angle NPO$	
$= 90$	$\angle NPO$ is a right angle.
$m\overset{\frown}{KON} = m\overset{\frown}{OK} + m\overset{\frown}{ON}$	Arc Addion Postulate
$180 = m\overset{\frown}{OK} + 90$	Substuon
$90 = m\overset{\frown}{OK}$	Subtract 90 from each side.

Measures of Arcs

B. In $\odot P$, $m\angle NPM = 46$, $\overline{PL}$ **bisects** $\angle KPM$, **and** $\overline{OP} \perp \overline{KN}$. **Find** $m\overset{\frown}{LM}$.



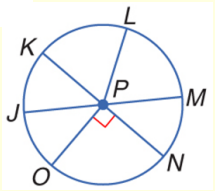
$m\overset{\frown}{LM} = \frac{1}{2}\overset{\frown}{KM}$ $\overline{PL}$ $\angle KPM$

$\overset{\frown}{KMN}$ is a semicircle.

$m\overset{\frown}{KM} + m\overset{\frown}{MN} = m\overset{\frown}{KMN}$	Arc Addion Postulate
$m\overset{\frown}{KM} + 46 = 180$	$m\overset{\frown}{MN} = m\angle NPM = 46$
$m\overset{\frown}{KM} = 134$	Subtract 46 from each side.
$m\overset{\frown}{LM} = \frac{1}{2}(134)$ or 67	

Measures of Arcs

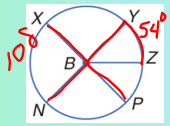
C. In $\odot P$, $m\angle NPM = 46$, $\overline{PL}$ **bisects** $\angle KPM$, **and** $\overline{OP} \perp \overline{KN}$. **Find** $m\overset{\frown}{JKO}$.



$m\angle NPM = 46$	
$m\angle KPJ = m\angle NPM$	Vercal angles are congruent.
$m\angle KPJ = 46$	Substuon.
$\angle KPJ + m\angle JPO = 90$	$\angle KPO$ is a right angle.
$46 + m\angle JPO = 90$	Substuon.
$m\angle JPO = 44$	Subtract 46 from each side.
$m\angle JPO = m\overset{\frown}{JO} = 44$	
$m\overset{\frown}{JO} + m\overset{\frown}{JKO} = 360$	
$44 + m\overset{\frown}{JKO} = 360$	Substuon.
$m\overset{\frown}{JKO} = 316$	Subtract 44 from each side.

Now, you try!

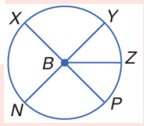
A. In $\odot B$, $\overline{XP}$ and $\overline{YN}$ are diameters, $m\angle XBN = 108$, and $\overline{BZ}$ bisects $\angle YBP$. Find $m\overset{\frown}{YZ}$.



$m\overset{\frown}{YZ} = 54^\circ$

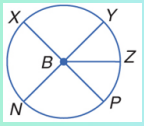
Now, you try again!

B. In $\odot B$, $\overline{XP}$ and $\overline{YN}$ are diameters, $m\angle XBN = 108$, and $\overline{BZ}$ bisects $\angle YBP$. Find $m\overset{\frown}{XY}$.



Come on! One more time! Don't give up on me!

C. In $\odot B$, $\overline{XP}$ and $\overline{YN}$ are diameters, $m\angle XBN = 108$, and $\overline{BZ}$ bisects $\angle YBP$. Find $m\overset{\frown}{XNZ}$.



$m\overset{\frown}{XNZ} = 234^\circ$

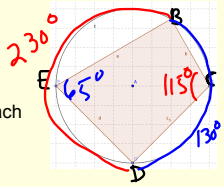
Quadrilaterals Inscribed in Circles

Facts about the Inscribed Quadrilateral

-All 4 angles add up to 360 degrees

-All 4 angles are inscribed angles

-Because the angles are inscribed, they will each equal exactly half of their corresponding Intercepted Arc.



$\angle E + \angle C = 180$

$\angle B + \angle D = 180$

A) Given Quadrilateral BCDE, and $m\angle E = 65^\circ$, find $m\overset{\frown}{BCD}$.

$m\angle E = \frac{1}{2} m\overset{\frown}{BCD}$

$m\angle B = \frac{1}{2} m\overset{\frown}{CDE}$

B)Use Properties of inscribed angles to show why $m\angle B + m\angle D = 180^\circ$.

$m\angle C = \frac{1}{2} m\overset{\frown}{DEB}$

$m\angle D = \frac{1}{2} m\overset{\frown}{EBC}$

$m\angle B + m\angle D = 180^\circ$

$m\angle B = \frac{1}{2} m\overset{\frown}{CDE}$ $m\angle D = \frac{1}{2} m\overset{\frown}{EBC}$

These 2 arcs make up the entire circle

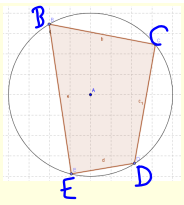
$\frac{1}{2} \cdot (m\overset{\frown}{CDE} + m\overset{\frown}{EBC} = 360^\circ)$

$\frac{1}{2} m\overset{\frown}{CDE} + \frac{1}{2} m\overset{\frown}{EBC} = 180^\circ$

$m\angle B + m\angle D = 180^\circ$

Quadrilaterals Inscribed in Circles

Now, you try!



A) Given Quadrilateral BCDE, and m ∠D = 65°, find m $\overset{\frown}{EBC}$.

B)Use Properties of inscribed angles to show why m ∠E + m ∠C = 180°.