

9.4 Area and Perimeter of Regular Polygons

OBJ: Calculate the area and perimeter of a regular polygon.

Perimeter of a Regular Polygon (Do not write this down)

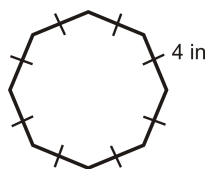
Recall that a regular polygon is a polygon with congruent sides and angles. In this section, we are only going to deal with regular polygons because they are the only polygons that have a consistent formula for area and perimeter. First, we will discuss the perimeter.

Recall that the perimeter of a square is 4 times the length of a side because each side is congruent. We can extend this concept to any regular polygon.



Perimeter of a Regular Polygon: If the length of a side is s and there are n sides in a regular polygon, then the perimeter is $P = ns$.

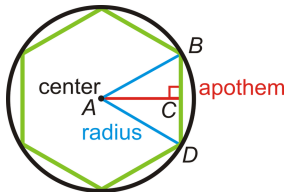
Example 1: What is the perimeter of a regular octagon with 4 inch sides?



Example 2: The perimeter of a regular heptagon is 35 cm. What is the length of each side?

Area of a Regular Polygon

All regular polygons can be inscribed in a circle. So, regular polygons have a center and radius, which are the center and radius of the circumscribed circle. Also like a circle, a regular polygon will have a central angle formed. In a regular polygon, however, the central angle is the angle formed by two radii drawn to consecutive vertices of the polygon.

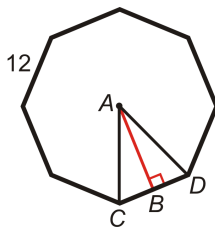


The central angle is $\angle BAD$. Also, notice that $\triangle BAD$ is an isosceles triangle. Every regular polygon with n sides is formed by n isosceles triangles. In a regular hexagon, the triangles are equilateral. The height of these isosceles triangles is called the apothem.

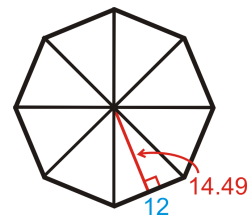
Apothem: A line segment drawn from the center of a regular polygon to the midpoint of one of its sides.

We could have also said that the apothem is perpendicular to the side it is drawn to. By the Isosceles Triangle Theorem, the apothem is the perpendicular bisector of the side of the regular polygon. The apothem is also the height, or altitude of the isosceles triangles.

Example 3: Find the length of the apothem in the regular octagon. Round your answer to the nearest hundredth.



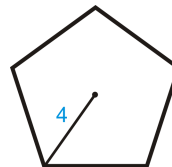
Example 4: Find the area of the regular octagon in Example 3.



Area of a Regular Polygon: If there are n sides with length s in a regular polygon and a is the apothem, then

$$A = \frac{1}{2}asn \text{ or } A = \frac{1}{2}aP, \text{ where } P \text{ is the perimeter.}$$

Example 5: Find the area of the regular polygon with radius 4.



Example 6: The area of a regular hexagon is $54\sqrt{3}$ and the perimeter is 36. Find the length of the sides and the apothem.