

Warm-up

1. We know that every circle is similar, is every sphere similar?
2. Find the volume of a sphere with a 12 in radius. Leave your answer in terms of π . 2304π
3. Find the volume of a sphere with a 3 in radius. Leave your answer in terms of π . 36π
4. Find the scale factor of the spheres from #2 and #3. Then find the ratio of the volumes and reduce it. What do you notice? $1/64 = 1/4$
5. Two squares have a scale factor of 2:3. What is the ratio of their areas? $4/9$
6. The smaller square from #5 has an area of 16 cm^2 . What is the area of the larger square? 36 cm^2
7. The ratio of the areas of two similar triangles is 1:25. The height of the larger triangle is 20 cm, what is the height of the smaller triangle? 4

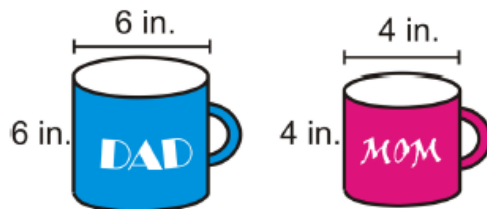
$$\frac{\sqrt{1}}{\sqrt{25}} = \frac{1}{5} = \frac{x}{20}$$

$$\frac{4}{9} = \frac{16}{x}$$

10.4 Similarity and volume

Know What? Your mom and dad have cylindrical coffee mugs with the dimensions to the right. Are the mugs similar? (You may ignore the handles.) If the mugs are similar, find the volume of each, the scale factor and the ratio of the volumes.

$$\frac{6}{4} = \frac{6}{4}$$



Recall that two shapes are similar if all the corresponding angles are congruent and the corresponding sides are proportional.

Similar Solids: Two solids are similar if and only if they are the same type of solid and their corresponding linear measures (radii, heights, base lengths, etc.) are proportional.

Example 1: Are the two rectangular prisms similar? How do you know?

$\frac{30}{45} = \frac{4}{6} = \frac{50}{75}$
 $\frac{3}{4.5} = \frac{5}{7.5}$
 $\frac{4}{6} = \frac{5}{7.5}$

The first rectangular prism has dimensions 3, 5, and 4. The second rectangular prism has dimensions 4.5, 7.5, and 6. The ratios of corresponding dimensions are $\frac{3}{4.5} = \frac{2}{3}$, $\frac{5}{7.5} = \frac{2}{3}$, and $\frac{4}{6} = \frac{2}{3}$. Since all ratios are equal, the two rectangular prisms are similar.

Example 2: Determine if the two triangular pyramids similar.

$\frac{6}{8} = \frac{8}{12} = \frac{12}{16}$
 $\frac{3}{4} = \frac{2}{3} = \frac{3}{4}$

The first triangular pyramid has a right-angled triangular base with legs of length 6 and 8, and a height of 12. The second triangular pyramid has a right-angled triangular base with legs of length 8 and 12, and a height of 16. The ratios of corresponding base legs are $\frac{6}{8} = \frac{3}{4}$ and $\frac{8}{12} = \frac{2}{3}$. The ratios of the heights are $\frac{12}{16} = \frac{3}{4}$. Since the ratios are not all equal, the two triangular pyramids are not similar.

Volumes of Similar Solids

Let's look at what we know about similar solids so far.

TABLE 2.3:

<i>Scale Factor</i>	<i>Ratios</i>	<i>Units</i>
<i>Ratio of the Surface Areas</i>	$\frac{a}{b}$	in, ft, cm, m, etc.
<i>Ratio of the Volumes</i>	$(\frac{a}{b})^2$	in^2, ft^2, cm^2, m^2 , etc.
	??	in^3, ft^3, cm^3, m^3 , etc.

$$\left(\frac{a}{b}\right)^3$$

Is there a pattern forming? If so what is it?

Example 7: Two spheres have radii in a ratio of 3:4. What is the ratio of their volumes?

$$\frac{3^3}{4^3} = \frac{27}{64}$$

Example 8: If the ratio of the volumes of two similar prisms is 125:8, what is their scale factor?

$$\frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$$

$$\sqrt[3]{\quad} \quad \sqrt[3]{\quad}$$

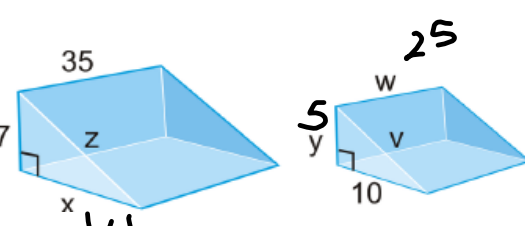
$$3 \sqrt[3]{125}$$

$$125 \sqrt[3]{3}$$

Example 9: Two similar right triangle prisms are below. If the ratio of the volumes is 343:125, find the missing sides in both figures.

$\frac{7}{5}$

$\frac{7}{y}$ $\frac{35}{w}$ $\frac{x}{10}$



$\frac{7}{5} = \frac{x}{10}$ $\frac{7}{5} = \frac{7}{y}$

Vol. $\frac{\sqrt[3]{343}}{\sqrt[3]{125}} = \frac{7}{5}$

$\frac{7}{5} = \frac{35}{w}$